Iver M

Rigid Body Notes

$p_t = p_{t-1} + \Delta t \cdot v_{t-1}$ movement integration

$\theta_t = \theta_{t-1} + \Delta t \cdot \omega_{t-1}$

- impulses (J) are instant change in momentum

$v = v + \frac{J}{m}$

scalar ~~vector~~ → • $J_{linear} = J$

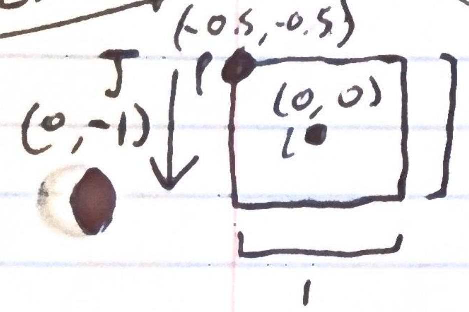
scalar → • $J_{angular} = J \times$ (point position − object center)

scalar cross product

Scalar Cross Product

- $A \times B = A.x \cdot B.y - A.y \cdot B.x$
- in 2D: 2D → scalar
- in 3D: 3D → 2D

Example



$J_{linear} = (0, -1)$

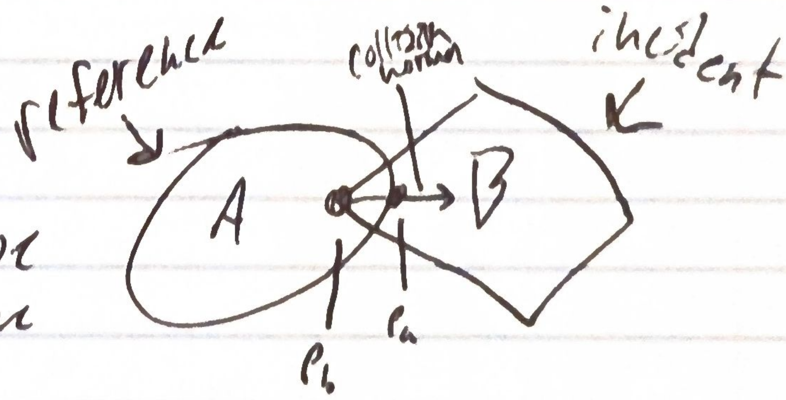
$J_{angular} = J \times (p - c)$

$(0, -1) \times (-0.5, -0.5)$

$0 \cdot -0.5 + 1 \cdot -0.5$

-0.5

Collisions



- p_b is the point on incident shape that is inside reference shape
- p_a is p_b projected onto reference edge
- collision normal is surface normal at collision, normal of reference edge
- collision point

$e = 0$: inelastic

$e = 1$: elastic

J equation

$$J = \frac{-(1+e)\,((v_a + (-\omega \cdot (p-c).y,\ \omega \cdot (p-c).x))}{\frac{1}{\tilde{M}_a} + \frac{1}{\tilde{M}_b}}$$

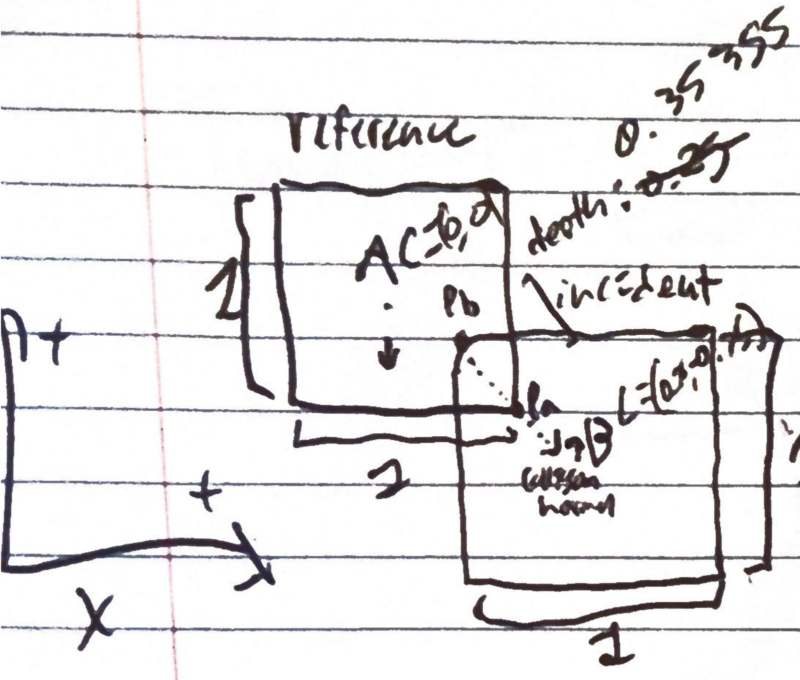
cross product

next is v_b, subtract it from v_a to get v_r

$$\tilde{M} = \left(\frac{p_n \cdot c}{|p_n c|}\right)^2 \times \frac{1}{I} + \frac{1}{M}$$

Iver M

Rigid Body Notes



A: $M = 1$ $I = 1$

$V = (0, -1)$ $\omega = 1$

$r = 0$ counter clockwise

B: $M = 1$ $I = 1$

$v = (0, 1)$ $\omega = -1$

$r = 0$ clockwise

Collision normal: $(0.7071, -0.7071)$

$e: 1$

P_a: ~~$\left(\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}\right)$~~ $(0.5, -0.5)$ P_b: ~~$\left(\frac{1}{4}\sqrt{2}, \frac{1}{4}\sqrt{2}\right)$~~ $(0.25, -0.25)$

P_a velocity $= [0, -1] + \left(-1 \overset{\text{cross}}{\times} ([0.5, -0.5])\right) =$
$[0, -1] + [0.5, -0.5] =$
$[0.5, -1.5]$

P_b velocity $= [0, 1] + (1 \times [0.5, -0.5]) =$
$[0, 1] + [0.5, 0.5] =$
$[0.5, 1.5]$

$V_n = \text{dot}(P_a - P_b, \text{normal}) =$
$\text{dot}\left([1, -3], \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]\right) = 1.41421356236$

Rigid Body Notes

Iver M

$J_{\cancel{numerator}} = \cancel{(1+1)\cdot(VP_a - VP_b) =}$

$$\cancel{\frac{-2\cdot 1.41421356236}{\frac{1}{\tilde{M}_a} + \frac{1}{\tilde{M}_b}}}$$

$$J = 1.41421356236$$

Effective Mass

$$\tilde{M}_a = \frac{1}{M_a} + \frac{d_a^2}{I_a} \qquad d_a = \text{magnitude}\big(\underset{0.5,\,-0.5}{(P-C)} \overset{\text{cross}}{\times} \underset{\frac{1}{\sqrt{2}},\,\frac{1}{\sqrt{2}}}{n}\big)$$

$$\frac{1}{1} + \left|-0.5\frac{1}{\sqrt{2}} - \left(-0.5\frac{1}{\sqrt{2}}\right)\right|$$

$\tilde{M}_a = 1$

$\tilde{M}_b = 1$

Ever M

Rigid Body Notes

- applying J

  - $V_a' = V_a + \text{Normal}\left(\frac{J}{M_a}\right)$
  - $\omega_a' = \omega_a + \frac{J((p-c) \times \text{normal})}{I_a}$ cross

  - same for B except J is negative

- $V_\Delta = \left[\frac{1}{\sqrt{2}} 1.414213562 36, \frac{1}{\sqrt{2}} 1.414213562 36\right]$

$$\omega_\Delta = \frac{1.414213562 36\left(0.5 \cdot -\frac{1}{\sqrt{2}} 1.414213562 36 - (-0.5 \cdot \frac{1}{\sqrt{2}} 1.414213562 36)\right)}{1}$$

$(0.5 \cdot -1 - (-0.5 \cdot 1))$

$-0.5 + 0.5 = 0$

$V_\Delta = [1, -1]$

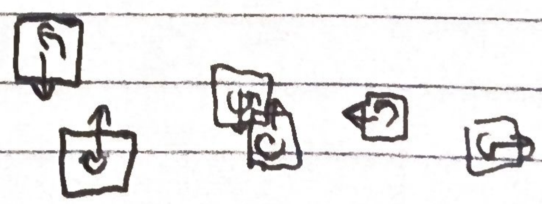
$\omega_\Delta = 0$

$\cancel{V_a' = [1, 2]}$ $\quad V_a' = [0, -1] - [1, -1] = [-1, 0]$

$\cancel{V_b' = [-1, 2]}$ $\quad V_b' = [0, 1] + [1, -1] = [1, 0]$

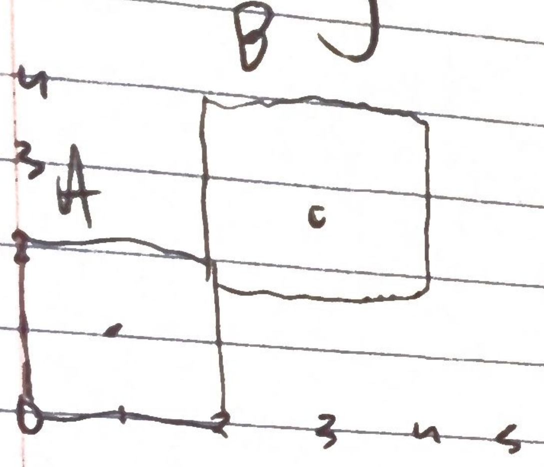
$\omega_a' = -1$

$\omega_b' = 1$



Rigid Body Notes

Ivv M



A: $p(1,1)$ $v=(0,0)$ $r=0$ $\omega=0$

B: $p=(3,3)$ $v=(-1,0)$ $r=0$ $\omega=0$

$p_a=(2,2)$ $p_b=(2,2)$

Normal: $(1,0)$

Point velocities

- no rotational velocity, so V_p reduces to shape V

$V_{pa}=[0,0]$

$V_{pb}=[-1,0]$

Normal Velocity Scalar

$V_n = \text{dot}(V_{pa}-V_{pb}, \text{normal}) = \text{dot}([1,0],[1,0]) = 1$

Dot Product

$a.x \cdot b.x + a.y \cdot b.y$

$$J = \frac{-(1+e)1}{\text{dist}^2(p_a,c_a)+\text{dist}^2(p_b,c_b)} = \frac{-2}{(1-2)^2+(1-2)^2+(3-2)^2+(3-2)^2} = \frac{-2}{4}$$

dist^2 - distance from center to collision point squared

$J = -0.5$

$V_\Delta = J[1,0] = [-0.5,0]$

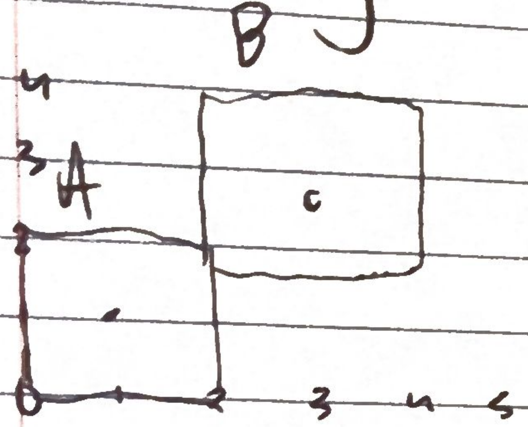
$\omega_{a\Delta} = J([-1,-1]\times[1,0]) = J(-1\cdot 0-(-1\cdot 1)) = J = -0.5$

$\omega_{b\Delta} = -J([1,1]\times[1,0]) = -J(1\cdot 0 - 1\cdot 1) = J = -0.5$

Luv M

Rigid Body Notes

$M=1$
$I=1$
$e=1$



A: $p(1,1)$ $v=(0,0)$ $r=0$ $\omega=0$
B: $p=(3,3)$ $v=(-1,0)$ $r=0$ $\omega=0$

$P_a=(2,2)$ $P_b=(2,2)$

Normal: $(1,0)$

Point velocities

- no rotational velocity, so V_p reduces to shape v

$V_{pa}=[0,0]$
$V_{pb}=[-1,0]$

Normal Velocity Scalar

$V_n = \text{dot}(V_{pa}-V_{pb}, \text{normal}) =$
$\text{dot}([1,0],[1,0]) =$
1

Dot Product

$a.x \cdot b.x + a.y \cdot b.y$

$$J = \frac{-(1+e)1}{\text{dist}^2(P_a, C_a) + \text{dist}^2(P_b, C_b)} =$$

$$\frac{-2}{(1-2)^2+(1-2)^2+(3-2)^2+(3-2)^2} =$$

dist^2 = distance from center to collision point squared

4

$J = -0.5$

$V_A = J[1,0] = [-0.5, 0]$
$\omega_{AS} = J([-1,-1] \times [1,0]) = J(-1\cdot 0 - (-1\cdot 1)) = J = -0.5$
$\omega_{bS} = -J([1,1] \times [1,0]) = -J(1\cdot 0 - 1\cdot 1) = J = -0.5$